"Now"

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Source: *Noûs*, May, 1968, Vol. 2, No. 2 (May, 1968), pp. 101-119
Published by: Wiley
Stable URL: https://www.jstor.org/stable/2214699

"*Now*"

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1. *The redundant present and the non-redundant "now"*. I have sometimes defended what might be called a "redundancy theory" of the present tense, analogous to the redundancy theory of truth propounded by Ramsey and Ayer.[1] I have argued that, whatever the proposition that *p* might be, the proposition that *it is (now) the case that p* is the very same proposition as the proposition that *p*. For example, the proposition that *it is now the case that I am sitting down* is the very same proposition as the proposition that *I am sitting down*; and equally, the proposition that *it is now the case that I have been* (or *will be*) *sitting down* is the very same proposition as the proposition that *I have been* (or *will be*) *sitting down*. For this reason, in developing symbolic systems of "tense logic", while I have introduced the form *Pp* for "It has been the case that *p*" and *Fp* for "It will be the case that *p*", I have introduced no analogous for "It *is* (now) the case that *p*", since I have taken the view that for this the plain *p* will do.

This position is, however, open to some criticism. I am not thinking of criticism from the point of view that the plain proposition *p* ought rather to be equated with the proposition that it is *timelessly* the case that *p*. This is so obviously not the case when we are considering *tensed* propositions that the point is not worth arguing about. I am thinking rather of criticism that could come as it were from *within* the enterprise of "tense logic". From this point of view it can hardly be denied that whatever the proposition that *p* might be, the proposition that it is now the case that *p entails and is entailed by* the proposition that *p*; or that any time at which

[1] A. N. Prior, *Time and Modality* (1957): 9-10; *Past, Present and Future* (1967): 8-10, 14-15; *Papers on Time and Tense* (1968): 17-23.

it is correct to assert either of these propositions is bound to be a time at which it is correct to assert the other. To say, however, that they are *one and the same* proposition, is to say rather more than this; and in particular it is to say that whatever is true of either proposition is true of the other, e.g. if either of them has been or will be the case then the other also has been or will be the case. But is it really true that *it will be the case that it is now the case that I am sitting down* if and only if *it will be the case that I am sitting down*? And is it really true that *it has been the case that it is now the case that I am sitting down* if and only if *it has been the case that I am sitting down*?

The language in which these last questions have been formulated is not ordinary idiomatic English, and this makes them difficult to answer. Still, let us repeat the first pair of propositions and stare at them for a moment:

(A) It will be the case that it is now the case that I am sitting down.

(B) It will be the case that I am sitting down.

The point at issue may be clearer if we replace these by a slightly more specific pair, say these:—

(C) It will be the case *tomorrow* that it is now the case that I am sitting down.

(D) It will be the case *tomorrow* that I am sitting down.

It would be natural to understand (D) as

(E) It will be the case tomorrow that I am *then* sitting down.

We would assert this truly at a given time if and only if we could truly assert the plain "I am sitting down" (or "I am now sitting down") *on the following day.* But it would not be at all natural so to understand (C). The most natural way to take (C) is to take it as being true at a given time if and only if the plain "I am sitting down" (or "I am now sitting down") is true *at that same time.* We are, in fact, inclined to regard the following propositoin as logically true:—

(F) If I am now sitting down, then it will always be the case that I am now sitting down,

though we are certainly not inclined to regard

(G) If I am now sitting down, then it will always be the case that I am *then* sitting down,

as being logically true; and as to

(H) If I am sitting down, then it will always be the case that I am sitting down,

we probably hesitate, not being quite sure whether to read it as (F) or as (G).

There is surely no need for prolonged agonising about all this. For (i), as far as English idiom goes, it seems clear that constructions involving the word "present" fit a redundancy theory fairly well, that ones involving the word "now" do not fit it at all well, and that ones involving the plain present tense or the plain "it is the case that" are in between. It will surely be agreed that what *will be present* is just what *will be*, and that what *has been present* is just what *has been*. "It will always be that I am *now* sitting down", on the other hand, does not ordinarily mean the same as "My sitting will always be present", or as "I will always be sitting down", as on a redundancy theory of "now" it ought to. And "It will always be that I am sitting down" and "It will always be the case that it is the case that I am sitting down" are just not quite ordinary English, and it is not clear whether an ordinary English speaker would equate them with "It will always be that I am now sitting down" or with "I will always be sitting down". And (ii), as far as a formalised logical language is concerned, it is clear that we can use its forms in any way we please, and in particular, we can certainly invent a language in which the form "It is the case that *p*", or even "It is now the case that *p*", is used to express whatever proposition is expressed by the plain *p*, and no other proposition than that. Can we not just leave it at that?

Not quite; there are several reasons why the dialogue between the constructors of formalisms and the recorders of idioms must be taken a little further. In the first place, the fondness of the former for the merely redundant present is not an arbitrary preference, and the reasons for it should be appreciated. It is not quite right to say that the formalised languages of most current tense-logics have *no* present tense. The present is, on the contrary, the understood tense of any proposition that has no other specific tensing; and it is therefore the tense of the "atomic propositions" or innermost kernels of all tensed constructions. There *has to be* such a tense if tense operators uniformly construct tensed propositions from tensed propositions; and moreover, this *has to be* a tense which every tensed proposition has even if it has no other. And it is natural for the tense-logician to go on to say, "I must and do have *this* present tense in my systems; surely I need no other."

He must do more than *say* this, however; he has to make a case for it—he has to show that whatever can be said with our idiomatic "now", the "now" for which $\varphi(p - \text{now})$ is *not* necessarily equivalent to $\varphi(p)$, can equally be said in his own language which contains no such operator. I believe that this can be done. How, I shall indicate in the next section, in which I closely follow Hector-Neri Castañeda. But until recently I would have gone further than this, and said that the formalist not only *can* do without the idiomatic "now" but *must* do without it—that our ordinary use of "now" has a certain fundamental disorderliness about it which makes it unamenable to formalisation (Section 3). Recently, however, I have been convinced to the contrary by Hans Kamp (Section 4), and have now myself produced an extension of tense-logic with a symbol corresponding fairly closely to the idiomatic "now" (Section 5).

2. *The elimination of the idiomatic "now"*. The essential point about the idiomatic "now" is that however oblique the context in which it occurs, the time it indicates is the time of utterance of the whole sentence. In "It will be the case tomorrow that my sitting down is present", the presentness referred is a presentness that will obtain tomorrow, i.e. at the time to which we are taken by the tensing prefix. But in "It will be the case tomorrow that I am sitting down now", the word "now" indicates the same time that it would indicate if it occurred in the principal clause—the time of utterance.

In an earlier paper I compared the word "now" at this point with the word "any", as opposed to "every". "Every" gives universality to its immediate context (e.g. "Not every man you meet is a liar" is a denial of "Every man you meet is a liar", this subordinate sentence being universal but the whole sentence particular), but "any", however obliquely it occurs, gives universality to the sentence as a whole (e.g. "Not any man you meet is a liar" is not a denial of "Any man you meet is a liar", but is, rather, equivalent to "Any man you meet isn't a liar").

Hector-Neri Castañeda has made a much more useful comparison of "now" with "I".[2] The primary function of "I" is self-reference, but in

(I) I think Brown thinks I can help him,

the second "I" does not indicate self-reference on the part of Brown, but rather on the part of the speaker of the whole sentence. Not, however, that that speaker has a thought that he would express by saying "I can help him"; oddly (considering the words used), *no* one is depicted in (I) as having such a thought. Brown would be depicted as having such a thought, and as referring to himself, in the sentence

(J) I think Brown thinks he can help me.

But not in (I). To express Brown's thought in (I) as Brown himself would express it, we would have to know Brown's way of referring to the speaker of the whole. The speaker of (I), however, does not profess to know what this is; what he does claim to know is something more indefinite that might be put thus:—

(K) I think that for some φ, (i) Brown thinks that (the only φ-er can help him), and (ii) I am the only φ-er.

Here the residual "I" of clause (ii) is self-referential, and clause (ii) does express the speaker's thought (*Brown's* thought being one that he would express by something of the form "The only φ-er can help *me*", this "me" being also self-referential).

"Now", Castañeda observes, is an adverbial analogue of the pronoun "I", and in

(L) It is now the case that I will later be glad that I am φ-ing now

the second "now" does not refer to the presentness of my φ-ing at the time of my gladness, but rather to its presentness at the time when the whole sentence is true. Not, however, that the speaker is said to have at the time of utterance a thought that he would express by saying "I am glad that I am φ-ing now"; indeed he is not said to have *that* thought at *any* time. He *would* have depicted himself as going to have that thought if he had said

(M) It is now the case that I will later be glad that I am φ-ing *then*

[2] H.-N. Castañeda, "Indicators and Quasi-indicators", *American Philosophical Quarterly*, IV (1967).

(which is related to (L) as (J) is to (I)). But not when he uses (L). To express the thought he depicts himself, when he uses (L), as going to have, we would need to know how he would refer, at the later moment when he is glad, to the moment when he uses (L). But in (L) itself he gives no indication of how he will later on think of that time. We may presume, however, that he will think of it as the only time at which some proposition or other is true (e.g. the time at which it is 6 o'clock GMT on April 10, 1968), so that (L) amounts to

> (N) It is now the case that for some proposition p which is true at one instant only, (i) it will be the case that I am glad it was the case that (p and I am φ-ing), and (ii) it is now the case that p.

Here both occurrences of "now" indicate the time of truth of the clauses in which they immediately occur, and nothing is lost if both of them are dropped.

In some such way as this, it seems to me, we *can* dispense with the non-redundant "now" in favour of the redundant one. In other words, the non-redundant "now" *is* non-redundant only in the sense that you cannot *just* erase it from a sentence and leave the sense of the whole the same; you can, however, erase it and get something with the same sense by altering the rest of the sentence somewhat.

But do we *have to* dispense with the non-redundant "now"? I shall indicate first why I formerly thought its introduction into tense-logic would have a quite explosive effect, and then why I now think it would not.

3. *The resistance of tense-logic to the idiomatic "now"*. We had best start by setting up a simple tense-logical system. We shall not take as primitive the earlier-mentioned operators F (for "It will be that") and P (for "It has been that"), but will define these in terms of G (for "It will always be that") and H (for "It has always been the case that"). Using N for "It is not the case that", we define Fp as $NGNp$ ("It will not always be the case that not p") and Pp as $NHNp$. Using Cpq for "If p then q", Kpq for "p and q", Apq for "p or q" and Epq for "p if and only if q", we assume some sufficient postulate-set for propositional calculus and then add the above definitions and the axioms

A1.1 $CGCpqCGpGq$ A1.2 $CHCpqCHpHq$

A2.1 $CPGpp$ A2.2 $CFHpp$.

Finally to the ordinary rules of substitution and detachment we add the following, for getting theorems from theorems:—

RG: If $\vdash \alpha$ then $\vdash G\alpha$

RH: If $\vdash \alpha$ then $\vdash H\alpha$

This system was called by the late E.J. Lemmon the system K_t.

It is possible to interpret the propositional variables of this system as if they were parts of *predicate* expressions in a first-order theory of the earlier-later relation. In this theory the individual variables, *a, b, c.* etc. are used for instants of time, the form *Uab* for "*a* is an earlier instant than *b*" and the form *Tap* for "It is true at *a* that *p*". Where complex formulae of tense logic are preceded by *Ta,* the resulting formulae can be equated to ones in which all the complexity has ben put *outside* whatever *Ta's* may be left. Our propositional-calculus operators *C* and *N* can be brought outside their *Ta's* by means of the equivalences

UT1. *ETaCpqCTapTaq*

UT2. *ETaNpNTap*

and if our other propositional-calculus operators are defined in terms of *C* and *N* we can easily derive from these two such further equivalences as *ETaKpqKTapTaq, ETaApqATapTaq,* etc. For our tense-operators G and *H,* using Πb for "for all *b*", we have

UT3. $ETaGp\Pi bCUabTbp$

UT4. $ETaHp\Pi bCUbaTbp$

From UT1-4 and the definitions of *F* and *P*, we can derive for these latter, by ordinary quantification theory, the equivalences $ETaFp\Sigma bKUabTbp$ and $ETaPp\Sigma bKUbaTbp$ (where Σb is for "For some *b*"). Ordinary quantification theory also suffices for the proof of various tense-logical formulae preceded by *Ta,* e.g. *TaCPGpp.* That is, we can prove in this system that anything of the form *CPGpp* is true at any arbitrary instant *a.* We can prove this, in fact, for all theorems of K_t; and if we assume nothing about the earlier-later relation *U* except UT1-4, we can *only* prove it for theorems of K_t. And UT1-4, we might well say, are not assumptions about the relation *U* at all; they are rather about the predicate-fragments *Cpq, Np, Gp* and *Hp,* and tell us under what conditions these are true at an instant *a.*

If we add further postulates which really are about the relation

U, e.g. the postulate $\Pi a \Sigma b Uab$, asserting that U has no last term, we may be able to attach to our arbitrary instant a the theorems of some richer tense-logic than K_t. The addition just mentioned in fact enables us to attach to any arbitrary instant a all the theorems derivable from K_t plus the further axiom *CGpFp*.

For the present, however, let us simply consider the system K_t, which is in a sense "minimal". It is as well to confine ourselves to this because I want to show that it is awkward to introduce into tense-logic an operator with the properties of the idiomatic "now", but if the tense-logic into which I introduce this operator is richer than K_t it is too easy to suggest that the trouble arises from my having made rash assumptions about time in the first place. So, for *G* and *H* (and *P* and *F*) we stick, in the meantime, to K_t.

Suppose we now write *Jp* for "It is *now* the case that *p*". For this operator we do need the postulates

J1. *CpJp* J2. *CJpp*,

i.e. we do wish to say "It is the case that *p* if and only if it is now the case that *p*". On the other hand, we do *not* wish to say *CFpFJp* and *CFJpFp*, "It will be the case that *p* if and only if it will be the case that it is *now* the case that *p*". Given K_t, however, these follow from J1 and J2. For RG will take us from J1 to *GCpJp*, and in K_t it is easy to prove *CGCpqCFpFq*, so from *GCpJp* we can go to *CFpFJp;* we reach *CFJpFp* from J2 by similar steps. Again, we want to say *CJpGJp*, "If it is now the case that *p* it will always be the case that it is now the case that *p*" (the generalisation of proposition (F) in the first section); but K_t suffices to take us from this, with J1 and J2, to *CpGp*, "If *p*, it will always be the case that *p*" (the generalisation of proposition (H) in the first section, interpreted in the sense of proposition (G), which we certainly do not want).

In fact, given *EJpp*, *EJpGJp* and *EJpHJp*, all of which seem desirable postulates for the idiomatic "now", and given K_t for *G* and *H*, we can prove *EpGp* and *EpHp;* indeed we can prove that *p*, *Gp*, *Hp*, *Fp* and *Pp* are all interchangeable in all tense-logical contexts, so that tenses are deprived of all usefulness. This result can, indeed, be tempered very slightly. A student of the idiomatic "now" might well decide that on reflection he *doesn't* want to commit himself to so much as *EJpGJp* and *EJpHJp*. For it will be evident on reflection that these postulates commit us not only to the standard properties of "now" but also to the assumptions that time had no beginning and will have no end. For the "always" in the "will always be" of our system is a *Boolean* "always", i.e. it is so used that *Gp*, "It will always be that *p*" comes out as vacuously true (as *Fp*, "It will be that *p*", comes out as vacuously false) if there is to be no future at all. Hence *CGJpJp* could be false if *GJp* is true not because it is now the case that *p* but merely because we are at the last moment of time. *CHJpJp* could be similarly false at time's first moment, if there were one. So the student of "now" who *didn't* want to commit himself to time's having no beginning and no end would weaken *EJpGJp* and *EJpHJp* to the corresponding one-way implications (*CJpGJp*, *CJpHJp*). This is enough, all the same, given the other postulates mentioned, to give us *CpGp* and *CpHp*, and from these *CFpp* and *CPpp*. Although these without their converses do not quite assert that there is no change at all, the only possible changes they would leave us with are those from the first moment of time (when everything of the form *Pp* is false) to later moments (when some things of this form are true) and to the last moment of time (when everything of the form *Fp* is false) from earlier moments (when some things of this form are true). This still makes tense-operators *almost* vacuous, and it seems to have this effect even with a minimum of assumptions about the character of the earlier-later relation, so a tense logician could very well be pardoned for refusing to admit such an operator as *J* into his system.

The trouble can be pin-pointed a little more closely. None of the four K_t axioms A1.1—A2.2 are rendered any less intuitively acceptable when formulae containing *J* are substituted for their variables, but RG and RH are a different matter. Take, e.g. J1, *CpJp*, "If *p* then it is now the case that *p*". This is alright, but the result of applying RG to it is *GCpJp*, "It will always be that if *p* then it is now the case that *p*", i.e. "It will always be that if it is *then* the case that *p* it is *now* the case that *p*". This, surely, is not alright at all. K_t, in other words, *could* accommodate *J* if we could drop RG and RM from it, or even if we could confine their operation to theorems in which *J* does not occur. This is still, however, a tall order if we are to think of K_t as embeddable in an earlier-later calculus in the usual way. For we can undoubtedly pass in such a calculus from $\vdash Ta\alpha$ to $\vdash TaG\alpha$ as follows:— From $\vdash Ta\alpha$ we obtain $\vdash Tb\alpha$ by substitution (α being unaffected by the substitution since it is a tense-logical formula and does not contain *a*), and from this we go to $\vdash CUabTb\alpha$ by *CpCqp*, from this to $\vdash \Pi bCUabTb\alpha$ by universal generalisation, and from this to $\vdash TaG\alpha$ by UT3. $\vdash TaH\alpha$ is obtained from $\vdash Ta\alpha$ analogously.

4. *A revised earlier-later calculus to accommodate "now"*. This last difficulty, however, is not insuperable. Hans Kamp devised in 1967[3] a consistent semantic interpretation for "now" which can be presented, with slight modifications, as a new sort of UT-calculus, in which T ties each tense-logical proposition not to one instant but to two, i.e. our basic form is not Tap but $Tabp$. The proposition p is related to the instants a and b in different ways; the essential difference is that the elimination of complexities from what is put after Tab may take us to other instants than a, but never to other instants than b. And wherever we may have been taken from a by operators like G and H, the one place to which we are always immediately taken by J is the instant b, i.e. the instant represented by the second argument of T. We might read the form $Tabp$ as "From b it is the case at a that p", and "From b it is the case at a that p—now" = "From b it is the case at b that p". So our basic equivalences are now

UT1. $ETabCpqCTabpTabq$

UT2. $ETabNpNTabp$

UT3. $ETabGp\Pi cCUacTcbp$

UT4. $ETabHp\Pi cCUcaTcbp$,

to which we add

UT5. $ETabJpTbbp$.

Substitution in UT1-4 will give us, it should be noted, the special cases $ETaaCpqCTaapTaaq$, $ETaaNpNTaap$, $ETaaGp\Pi cCUacTcap$ and $ETaaHp\Pi cCUcaTcap$. And we seek for those tense-logic formulae which, when preceded by any arbitrary duplicated prefix Taa, are provable in the new calculus. It will be found that these include J1, J2, $CJpGJp$ and all four of the *axioms* of K_t. But we cannot pass from $\vdash Taa\alpha$ to $\vdash TaaG\alpha$ by echoing the moves that formerly took us from $\vdash Ta\alpha$ to $\vdash TaG\alpha$. For we cannot take the first step, which would be from $\vdash Taa\alpha$ to $\vdash Tca\alpha$—that half-substitution is not legitimate. All the same, for any α *not involving* J for which we can prove $\vdash Taa\alpha$, we can also prove $\vdash Tca\alpha$ (by similar steps—apart from J, it does not matter whether the second instant is the same as the first or not), and so $\vdash TaaG\alpha$ and $\vdash TaaH\alpha$. We therefore have not only all the axioms but all the theorems of K_t, and also all

[3] In a multilith, "The treatment of 'now' as a 1-place sentential operator"; circulated at the University of California in Los Angeles.

substitutions in such theorems, including ones in which *J* occurs. But we do not have the *G*-ings and *H*-ings of J1 and J2.

Given this basis, what other tense-logical formulae (in *G, H* and *J*) can we prove to be attachable to any arbitrary *Taa*? What we want is of course a set of postulates for tense-logic in *G, H* and *J* from which all such formulae are deducible. I hope that what I say later (in Section 5, remarks (iv) to (vi)) will throw some light on this problem, but I want now to tackle what turns out to be a simpler one, which arises when we enrich our tense-logic with the further function "*p* at all times". We may write this as *Lp*, and lay down the characterising equivalence.

UT6. *ETabLp*Π*cTcbLp*

(from which we get *ETaaLp*Π*bTbaLp*). This addition immediately gives us, as preceded by any arbitrary *Taa*, whatever is derivable from the following postulates:—

RL: If $\vdash \alpha$ then $\vdash L\alpha$, provided that α does not contain *J*

L1. *CLpp* L2. *CLCpqCLpLq*

L3. *CNLpLNLp*

L4. *CLpGp* L5. *CLpHp*

And we can now ask: what postulates in *G, H, J* and *L* (not just *G, H* and *J*) will yield all those formulae which, preceded by an arbitrary *Taa*, are provable from quantification theory and UT1-6? I shall answer this piecemeal, noting now that (i) RL and L1-3 (subjoined to propositional calculus) are known to suffice for the pure *L* portion of the required calculus, which is the modal system S5. (ii) From RL, L4 and L5 we can derive the modified RG and RH that we need for the *G-H* portion of our calculus. (iii) For the full *L-G-H* portion we need the above RL and L1-5, together with the four K_t axioms A1.1 - 2.2. All that remains is to add the postulates for *J*, but before doing that I must digress again.

Kamp's is not the only way of modifying the UT calculus to obtain one that will accommodate *J;* though it is, so far as I know, the first solution offered to that problem. We might instead keep the simple form *Tap*, and the postulates UT1-4 in their original forms, but introduce a *constant,* say *n,* for a particular instant, with the following equivalences for *J* and *L*:

UT5. *ETaJpTnp* UT6. ETaLpΠbTbp

We then seek for those tense-logical formulae which can be proved attachable either to any arbitrary instant *a*, and so to *n*, or just to *n*. (Note that we cannot pass from $\vdash T n \alpha$ to $\vdash T b \alpha$, and so to $\vdash \Pi b T b \alpha$, because *n* is a constant). These can easily be seen to be exactly the same formulae as those which are attachable in Kamp's revision of the UT calculus either to any arbitrary *Tab*, and so to those in which the two instant-arguments are the same, or just to these latter. The use of a constant, however, has the advantage of having been tried before for another purpose. It was into just such a modification of the "property calculus" that C.A. Meredith, round about 1953, embedded his modal logic with a contingent constant *n* for "the world", in the sense of "everything that is the case". And we do know how to axiomatise that.[4]

5. *A tense-logic with "now"*. We may begin by incorporating *n* not only into our UT calculus as an instant-constant but also into our tense-logic itself as a *propositional* constant, standing for some proposition (any proposition) which is true at this moment only. Since our calculus is designed to yield as theorems whatever is formulable in the system's symbolism and is true at this moment (apart from truths arising from special properties of the earlier-later relation), the new symbol gives us also a new axiom, viz

A3. *n*.

Let us now bring from the UT calculus into the tense-logic itself not only *n* but also the variables *a, b, c* etc., each of which may stand for any proposition which is true at one instant and at one instant only. For these we have the two axioms

A4. *NLNa* A5. *ALCapLCaNp.*

Using *Mp* ("*p* at some time") as short for *NLNp*, we can abridge A4 to *Ma*. Finally, we define *Jp* either as *MKnp* ("At some time, *p* true together with NOW") or as *LCnp* ("*p* true at all times at which NOW"), alternatives which the uniqueness of NOW makes equivalent; and we enlarge the restriction on RL to "provided that α does not contain *J* or *n*".

The postulates RL (modified), L1-5 and A1.1-5, with the definition of *J*, will I think yield all the theorems in *J* that we want;

[4] C. A. Meredith and A. N. Prior, "Modal Logic with Functorial Variables and a Contingent Constant", *Notre Dame Journal of Formal Logic*, VI (1965): 99-109.

and the definition of "now" as expressing contemporaneity with some unspecified proposition which is true only at the time of utterance nicely formalises Castañeda's explanation of the use of "now" in oblique contexts. It may be felt, however, that this system is too much of a hybrid between a UT calculus and a tense logic. So let us hedge no more: we drop A3-5 and the proposed definition of *J*, and give postulates for an undefined *J* to be added to RL, L1-5 and A1.1 - 2.2 of K_t; namely the following:—

J1. *CpJp*	J3. *LCLpJp*	J5. *LCJNpNJp*
J2. *CJpp*	J4. *LCJpLJp*	J6. *LCNJpJNp*
	J7. *LCJCpqCJpJq*.	

That these postulates are complete, is a conjecture. The following results, however, are certain:

(i) From RL, L1-3, and J1-7 we can prove all of the following permanent equivalences: *LEJJpJp, LELJpJp, LELpJLp, LENJpJNp, LECJpJqJCpq*. These enable us, within this sub-calculus (i.e. the part without *G* or *H*), when all the variables in a formula fall within the scope of an *L* or a *J*, to equate the whole with a formula in which the only *J* occurs right at the beginning, and there we can strengthen it to *LJ* by J4. So RL applies to formulae of this sort as well as to ones with no *J*'s in them at all. But when we bring *G* and *H* in we do not have either *LEGJpJp* or *LEHJpJp*, but only the weaker *LCJpGJp* and *LCJpHJp* (from J4, L4 and L5); to get the converses, as we have noted earlier, we need to add tense-logical postulates (say *CGpFp* and *CHpPp*) which assume that time has neither a first nor a last instant. Only when such additions are made can we extend RL to *all* formulae of the calculus in which every variable falls within the scope of an *L* or a *J* (without them we have, e.g. *CGJpJGJp*, from J1, but not *LCGJpJGJp*).

(ii) Given RL, L1-3 and A3-5, all of J1-7 are deducible from the definition of *Jp* as *LCnp*, or from *LEJpLCnp*, and conversely, *LEJpLCnp* from J1-7. (The Castañeda reduction is thus possible but optional). We still have this result if instant-*variables* are dropped and A4 and 5 replaced by *CpLCnp*.

(iii) I have pointed out elsewhere[5] that we can bring the whole UT symbolism within tense-logic by defining *Tap* as *LCap*

[5] E.g. in Paper XI of *Papers on Time and Tense* (1968).

(or *MKap*) and *Uab* as *TaFb*. If we assume these definitions, quantification over the variables standing for "instant-propositions", and the "bridging postulates" A3-A5, then (a) UT1, 2 and 6 become provable from RL and L1-3, as well as *vice versa;* (b) UT3 and 4 become provable also when we add L4, L5 and the four K_t axioms A1.1-2.2, as these do when we add UT3 and 4; and (c) UT5 becomes provable when we add either *LEJpLCnp* or J1-7 to the pure *L* group, as these do when we add UT5 to it. Results (a) and (b) also hold if we replace A3 (*n*) by Σaa, which of course follows from it.

I add some results in pure *G-H-J*.

(iv) Suppose we introduce the *G-H* forms L^0p, L^1p, L^2p, etc. by the following definitions:—

$$L^0p = p$$
$$L^{n+1}p = KKL^npGL^npHL^np$$

(hence L^1p is *KKpGpHp*; L^2p is $KKL^1pGL^1pHL^1p$, i.e.

$$KK(KpGpHp)G(KpGpHp)H(KpGpHp),$$

which in K_t is equivalent to

$$KKKKKKpGpHpGGpGHpHGpHHp;$$

and so on. In general L^n contains all possible *H-G* sequences with up to *n* members). This notation enables us to represent and assert an infinity of tense-logical formulae at once; thus, e.g. CL^npp is a scheme that covers *Cpp* ($= CL^0pp$), *CKKpGpHpp* ($= CL^npp$), and so on. If α is a theorem of K_t, so are all formulae $L^n\alpha$ (cf. RL); and we have as K_t theorems all formulae of the forms

CL^npp (cf. L1); $CL^nCpqCL^npL^nq$ (cf. L2);

and for $n \geqslant 1$,

CL^npGp (cf. L4); CL^npHp (cf. L5);

but (except where $n = 0$) we do not have $CNL^npL^nNL^np$, the analogue of L3, but only the weaker $CNpL^nNL^np$ (this corresponds to the weakening, in modal logic, of S5 to the "Brouwersche" system). UT1-5 also verify (as attachable to *Ta*) all the L^n analogues of J4-7, but not that of J3, L^nCL^npJp (this, again, we have for $n = 0$ only; though we also have CL^npJp, without the initial L^n, from L^n1 and J1). Nor, it may be added, are the L^n analogues of

L3 and J3 provable in the richer system in which K_t is supplemented by the L and J postulates in their original forms; though in this richer system we can prove all the L^n analogues of J4-7. Whether these last could be proved from a finite set of axioms in G, H and J, verified by UT1-5, I do not know. They can be proved easily enough, however, and the L^n analogues of L3 and J3 too, if we drop the condition "verified by UT1-5".

(v) If we use the form *Iab* for "*a* is the same instant as *b*" we can (if we wish) add to UT1-5 the following special condition on *U*:—

UT7. *CAUacUcaCAUbcUcbAAUabUbaIab.*

This states approximately that every instant is either earlier or later than every other instant in the same time-system. With this addition our U-calculus will verify, as attachable to any *Ta*, not only K_t but all theorems derivable in K_t with the added axiom

A6. *CKKpGpHpKKKGGpGHpHGpHHp.*

(Conversely, adding A6 to RL, L1-5, A1.1-5, and the definitions of *T* and *U*, and of *Iab* as*LEab*, enables us to prove UT7). A6 can easily be shown equivalent to CL^1pL^2p, which by substitution yields $CL^1L^1pL^2L^1p$, and so CL^2pL^3p, and so by syllogism CL^1pL^3p, and by repetition of these steps CL^1pL^np for any n. This means that if we lay down J4-7 with L^1 for L we can now derive J4-7 with any L^n for L. The addition of A6 to K_t also enables us to prove L3 (and so all of RL and L1-5) with L^1 for L. It is the weakest addition to K_t that will do this. (Given K_t, A6 implies but is not implied by $CL^1pKGHpHGp$, which reflects time's non-branching, and is implied by but does not imply that $\vdash$ *CGpGGp*, which reflects *U*'s transitivity.) The set UT1-7 is still not strong enough to verify J3 with L^1 for L, let alone with any L^n you please for L.

(vi) If we add to UT1-5, instead of UT7, the rather stronger

UT8. *AAUabUbaIab.*

which asserts straightforwardly that every instant is earlier or later than every other (ruling out the possibility of distinct time-systems), we can prove *ETaKKpGpHp*Π*bTbp*, equating "It is and always will be and always has been the case that *p*" (true at some *a*) with "*p* at all times", and making *L* a superfluous symbol. The addition of CL^1pLp, or of the definition of L as L^1, to RL, L1-5, A1.1-5 and the definitions of *T*, *U* and *I*, yields UT6-8 as well as UT1-4; and everything that can be proved of L from UT1-6, including Ta(L3) and Ta(J3), can be proved of the purely G-H operator L^1 from UT1-5 + UT 8.

Kamp has raised this further question: What postulates will suffice for G-H-J if we assume that U has the properties of the relation "less than" among real numbers? I suspect that the answer is: To K_t (with restricted RG, RH) and J1-7 (with L defined as L^1) add axioms for U's transitivity (e.g. $CGpGGp$) and for time's linearity (e.g. $CL^1pKGHpHGp$), infinity (e.g. McCall's $CpKPFpFPp$[6]) and continuity (e.g. Bull's $CGGpGp$, $CHGCGpPGpCGpHp$ and $CGHCHpFHpCHpGp$[7]).

It can now be seen, also, that the Meredith paper cited at the end of Section 4 has more to our purpose than what we first found there, the constant *n*. Meredith's propositional-calculus primitives are C and an impossible (or permanently false) constant 0 ($Np = Cp0$), and he has not only propositional variables but one (δ) for one-place connectives. He has these laws for L, n and J:—

M1. $LC\delta CCp0Cqr\delta CCrpCqp$

M2. $CLpC\delta Cpq\delta q$

M3. $C\delta 0C\delta C00\delta Lp$

M4. n (= A3)

M5. $CpLCnp$

M6. $CpJp$ (= J1)

M7. $CJpp$ (= J2)

M8. $C\delta JCpq\delta CJpJq$

M9. $C\delta J0\delta 0$

M10. $C\delta 0C\delta C00\delta Jp$

Here M1-3, with substitution (for each sort of variable) and detachment, are equivalent to the whole of S5 (including propositional calculus) + $CLEpqC\delta p\delta q$. For n Meredith also has $CLnp$, which denies that ours is the only possible world, or in tense-logic that now is the only instant; we omit it as out of place in a *minimal* system. He merely mentions M6-10 at the end of the article as being true of Jp defined as $LCnp$; but as a group there is more interest

[6] S. McCall, review of *Past, Present and Future* in *Dialogue*.

[7] R. A. Bull, "An Algebraic Study of Tense Logics with Linear Time", *Journal of Symbolic Logic*.

to them than that. Given M1-5, they yield as well as being yielded by *LEJpLCnp* (or *CδJpδLCnp*); and given just M1-3, they yield and are yielded by our own J1-7.

Appendix:—

Proofs of equivalences from RL, L1-3, J1-7:

1. *LJCpJp* (J1, J1, J4)
2. *LCJpJJp* (L2, J7, 1)
3. *LJCJpp* (J2, J1, J4)
4. *LCJJpJp* (L2, J7, 3)
*5. *LEJJpJp* (2, 4)
6. *LCLpp* (L1, RL)
*7. *LELJpJp* (6 *p/Jp*; J4)
8. *LCNLpJNLp* (L3, RL; J3)
9. *LCNLpNJLp* (8, J5)
10. *LCJLpLp* (9, transp.)
11. *LCLLpJLp* (J3)
12. *LCLpJLp* (11; *LCLpLLp* from S5)
*13. *LELpJLp* (10, 12)
*14. *LENJpJNp* (J5, J6)
15. *LCJNCpqJKpNq* (*CNCpqKpNq*, J1, J4, J7)
16. *LCJKpNqJp* (*CKpqp*, J1, J4, J7)
17. *LCJKpNqJNq* (*CKpqq*, J1, J4, J7)
18. *LCJKpNqNJq* (17, J5)
19. *LCJKpNqKJpNJq* (16, 18)
20. *LCNJCpqNCJpJq* (J6, 15, 19, *LCKpNqNCpq*)
21. *LCCJpJqJCpq* (20, transp.)
*22. *LECJpJqJCpq* (21, J7)

Add A3 and A5 to get

23. *CLCnpp* (A3, *CpCLCpqq*)
24. *LJCLCnpp* (23, J1, J4)
25. *LCJLCnpJp* (L2, J7, 24)
26. *LCLCnpJp* (25, 12)
27. *CpNLCnNp* (23, *p/Np*, transp.)
28. *CpLCnp* (27, A5) (= M5)
29. *LCJpLCnp* (28, J1, J4, J7, 10)
*30. *LEJpLCnp* (26, 29)

Add M1-3 (from which Ext: *CLEpqCδpδq*) and Df. *N* to get

31. *LCJCp0CJp0* (J5, Df. *N*)
32. *LJC00* (*Cpp*, J1, J4)

33. $LCJ00$ (31, 32)
34. $LC0J0$ ($C0p$, RL)
35. $LEJ00$ (33, 34)
*36. $C\delta JCpq\delta CJpJq$ (J7, Ext.) (= M8)
*37. $C\delta J0\delta 0$ (35, Ext.) (= M9)
38. $C\delta LJp\delta Jp$ (7, Ext.)
*39. $C\delta 0C\delta C00\delta Jp$ (M3 p/Jp; 38) (= M10)

Proofs from M1-3, 6-10:

40. $LEJCpqJCpq$ ($LEpp$)
*41. $LEJCpqCJpJq$ (M8, 40) (gives J7)
42. $LEJCp0CJpJ0$ (41)
43. $LEJ00$ ($LEJ0J0$, M7)
44. $LEJCp0CJp0$ (42, 43)
*45. $LEJNpNJp$ (44, Df. N) (= J5, J6)
*46. $LECJ0J0C00$ ($LECppCqq$)
47. $LEJC00C00$ (41, 46)
*48. $LEJLpLp$ (M3, 43, 47) (gives J3)
*49. $LELJpJp$ (M10; $LEL00$ and $LELC00C00$ from M1-3) (gives J4)

Proof from RL, L1-3, A4, 30, Df. T:—

50. $CLCnpLCaLCnp$ ($CLpLCqLp$, in S5)
51. $CLCaLCnpCMaMLCnp$ ($CLCpqCMpMq$, in S5)
52. $CLCaLCnpMLCnp$ (51, A4)
53. $CLCaLCnpLCnp$ (52, $CMLpLp$)
54. $ELCaLCnpLCnp$ (50, 53)
*55. $ETaJpTnp$ (54, 30, Df. T) (= UT5)

From A4-6 we obtain Ta(A6), from which with UT1-4, and the definitions of T,U and I we may prove UT7 thus (employing the lemma $ETaFbTbPa$, which I have proved in *Past, Present and Future*):—

C(1)$AUacUca$
C(2)$AubcUcb$
C(3)$NUab$ (= $NTaFb$ = $TaNFb$ = $TaGNb$)
C(4)$NUba$ (= $NTaPb$ = $TaNPb$ = $TaHNb$)
K(5)$AAAKUacUbcKUacUcbKUcaUbcKUcaUbc$ (1, 2, p.c.)
K(6)$CTaNbCTaGNbCTaHNb$
-$KKKTaGGNbTaGHNbTaHGNbTaHHNb$ (Ta(A6))
K(7)$CTaNbKKKTaGGNbTaGHNbTaHGNbTaHHNb$ (3, 4, 6)
K(8)$CTaNbKKKCKUacUbcTbNbCKUacUcbTbNb$
-$CKUcaUbcTbNbCKUcaUbcTbNb$ (7, UT3, UT4, instantiations from $\Pi cCUac\Pi dCUcdTdNb$, etc.)
K(9)$CTaNbTbNb$ (5, 8, p.c.)

$K(10)NTaNb$ (9, Df. T, $CLCpNpNMp$, A4)
$K(11)NTbNa$ (10, Df. T, LC — transp.)
$K(12)KTabTba$ (10, 11, UT2)
(13)Iab (12, Df. T, Df. I)

Proofs of other results noted in the text (where not already to be found in *Past, Present and Future* or *Papers on Time and Tense*) may be left to the reader.